equilibrium, disturbed at the surface by the effect of the winds. The conditions are nearly identical. In the Baltic, easterly winds and currents force the waters into the North Sea, raise the level of the Great Belt, and give rise to an undercurrent from the North Sea into the Baltic. Here we assume that the waters, which the equatorial currents succeed in piling up in the western part of the Caribbean Sea, seek a passage through the Yucatan Strait into the Gulf of Mexico; that in this passage the level stands higher than on either side, and that the water forced into the Gulf of Mexico raises its level over that of the eastern portion of the Caribbean Sea. Since equilibrium cannot be restored at the surface, it is done by an undercurrent from the Gulf. Somewhat different from these conditions are those which are supposed to govern the flow of the Gulf Stream from the Gulf of Mexico into the Atlantic. Pillsbury's current observations in the Strait of Florida show that in the narrow parts the current touches bottom. There remains little doubt at present that the Gulf Stream owes its origin to the difference of level between the Gulf of Mexico and the Atlantic. Recent precise leveling, by the Coast and Geodetic Survey, indicates that between the mean level of the ocean at St. Augustine, on the eastern coast of Florida, and that of the Gulf at Cedar Keys, on the western, there exists a difference of nine-tenths of a foot. Some surprise might be expressed that such an insignificant difference should be able to set such a powerful stream into motion. But if we assume the Gulf to represent a basin, and the Strait of Florida a narrow orifice by which it communicates with the Ocean and apply Torricelli's theorem, neglecting friction, we obtain the velocity of $v = \sqrt{2g \times 0.9} = 7.6$ feet per second, which is not very greatly in excess of the average velocity of the Gulf Stream in the most contracted portion of the Strait.

Some authors speak of the impulse of the Gulf Stream carrying its waters against the western coast of Europe, and producing a higher level there than exists on the eastern coast of North America. Whatever impulse the Gulf Stream possesses is due to its higher level, and I cannot comprehend how such an impulse can make it ascend an inclined plane. What is meant, I presume, is the Gulf Stream drift, the motive power of which is the prevailing westerly winds of the North Atlantic. It is generally supposed that this Gulf Stream drift is compensated for by an undercurrent setting from the western shores of Europe in a southwesterly direction.

A. LINDENKOHL.

OBSERVATIONS ON RHYTHMIC ACTION.

Two entirely different forms of regularly repeated action are to be distinguished. In one form the subject is left free to repeat the movement at any interval he may choose. This includes such activities as walking, running, rowing, beating time, and so on. A typical experiment is performed by taking the lever of a Marey tambour between thumb and index finger and moving the arm repeatedly up and down; the recording tambour writes on the drum the curve of movement. Another experiment consists in having the subject tap on a telegraph key or on a noiseless key and recording the time on the drum by sparks or markers. Other experiments may be made with an orchestra leader's baton having a contact at the extreme end, with a heel contact on a shoe, with dumb-bells in an electric circuit, and so on. For this form of action I have been able to devise no better name than 'free rhythmic action.'

In contrast with this there is what may be called 'regulated rhythmic action.' This is found in such activities as marching in time to drum-beats, dancing to music, playing in time to a metronome, and so on. A

typical experiment is that of tapping on a key in time to a sounder-click, the movement of the click and that of the movement of the finger being registered on a drum.

Regulated rhythmic action differs from free rhythmic action mainly in a judgment on the part of the subject concerning the coincidence of his movements with the sound heard (or light seen, etc.). This statement, if true, at once brushes aside all physiological theories of regulated rhythmic action. One of these theories is based on the assumption (Ewald) that the labyrinth of the ear contains the tonus-organ for the muscles of the body. It asserts that vibrations arriving in the internal ear affect the whole contents including the organ for the perception of sound and the tonus-organ. Thus, sudden sounds like drum-beats or emphasized notes would stimulate the tonus-organ in unison, whereby corresponding impulses would be sent to the muscles. This theory has very much in its favor. It is undoubtedly true that such impulses are sent to the muscles. Thus at every loud stroke of a pencil on the desk I can feel a resulting contraction in the ear which I am inclined to attribute to the *M. tensor tympani*. Likewise a series of drum-beats or the emphasized tones in martial or dance music seem to produce twitchings in the legs. Féré has observed that, in the case of a hysterical person exerting the maximum pressure on a dynamometer, the strokes of a gong are regularly followed by sudden increased exertions. Nevertheless, these twitchings are not the origin of the movements in regulated rhythmic action. For many years I have observed that most persons regularly beat time just before the signal occurs; that is, the act is executed before the sound is produced. Records of such persons have been published (*e. g.*, New Psychology, p. 182), but their application to the invalidation of the tonus-theory was first suggested by Mr. Ishiro Miyake. This does not exclude the use of muscle sensations, derived from tonus-twitches, in correcting movements in regulated rhythmic action, although they presumably play a small or negligible part as compared with sounds.

Another argument in favor of the subjective nature of regulated rhythmic action is found in the beginning of each experiment on a rhythm of a new period; the subject is quite at loss for a few beats and can tap only spasmodically until he obtains a subjective judgment of the period. If the tonus-theory were correct, he should tap just as regularly at the start as afterward.

The conclusion seems justified that regulated rhythmic action is a modified free rhythmic action, whereby the subject repeats an act at what he considers regular intervals, and constantly changes these intervals to coincide with objective sounds which he accepts as objectively regular.

In free rhythmic action there is one interval which on a given occasion is easiest of execution by the subject. This interval is continually changing with practice, fatigue, time of day, general health, external conditions of resistance, and so on.

"It has long been known that in such rhythmic movements as walking, running, etc., a certain frequency in the repetition of the movement is most favorable to the accomplishment of the most work. Thus, to go to the greatest distance in steady traveling day by day the horse or the bicyclist must move his limbs with a certain frequency; not too fast, otherwise fatigue cuts short the journey, and not too slow, otherwise the journey is made unnecessarily short. This frequency is a particular one for each individual and for each condition in which he is found. Any deviation from this particular frequency diminishes the final result."

It is also a well-known fact that one rate of work in nearly every line is peculiar to

each person for each occasion, and that each person has his peculiar range within which he varies. Too short or too long a period between movements is more tiring than the natural one in walking, running, rowing, bicycling, and so on.

It is highly desirable to get some definite measurement of the difficulty of a free rhythmical action. This cannot well be done by any of the methods applicable to the force or quickness of act, but it may be accomplished in the following manner:

As a measure of the irregularity in a voluntary act we may use the probable error. When a series of measureable acts are performed they will differ from one another, if the unit of measurement is fine enough. Thus, let $x_1, x_2, \cdots, x_n$ be successive intervals of time marked off by a subject beating time, or walking, or running, at the rate he instinctively takes. The average of the measurements,

$$a = \frac{x_1 + x_2 + \cdots + x_n}{n},$$

can be considered to give the period of natural rhythm under the circumstances. The amount of irregularity in the measurements is to be computed according to the well-known formula:

$$p = \sqrt{\frac{v_1^2 + v_1^2 + \cdots + v_n}{n - 1}}$$

where $v_1 = x_1 - a,\ v_2 = x_2 - a,\ \cdots,\ v_n = x_n - a$. The quantity p is known as the 'probable error,' or the 'probable deviation.' The quantity

$$r = \frac{p}{a},$$

the 'relative probable error,' expresses the probable error as a fraction of the average.

If all errors in the apparatus and the external surroundings have been made negligible, this 'probable error' is a personal quantity, a characteristic of the irregularity of the subject in action. If, as may be readily done, the fluctuations in the action of the limbs of the subject be reduced to a negligible amount, this probable error becomes a central, or subjective, or psychological, quantity. Strange as it may appear, psychologists have never understood the nature and the possibilities of the probable error (or of the related quantities, 'average deviation,' 'mean error,' etc.). In psychological measurements it is—when external sources of fluctuation are rendered negligible—an expression for the irregularity of the subject's mental processes. Nervous or excitable people invariably have large relative probable errors; phlegmatic people have small ones.

Thus a person with a probable error of 25% in simple reaction time will invariably have a large error in tapping on a telegraph key, in squeezing a dynamometer, and so on. I have repeatedly verified this in groups of students passing through a series of exercises in psychological measurements. I do not believe it going too far to use the probable error as a *measure* of a person's irregularity. This is equivalent to asserting that a person with a probable error twice as large as another's is twice as irregular, or that if a person's probable error in beating time at one interval is r_1 and at another interval r_2, his irregularity is r_1 times as great in the second case as in the first. This concept is analogous to that of precision in measurements. We might use the reciprocal of the probable error as a measure of regularity. The positive concept, however, is in most minds the deviation, variation or irregularity, and not the lack of deviation, the non-variability, or the regularity. In the case of the word 'irregularity' the negative word is applied to a concept that is naturally positive in the average mind.

The irregularity in an act is a good expression of its difficulty. Thus, if a person beating time at the interval T has an ir-

regularity measured by the probable error P and at the interval t a probable error p, it seems justifiable to say that the interval t is $\frac{p}{P}$ times as difficult as T. If T is the natural interval selected by the subject, then the artificial interval t would be more difficult than T, and we should measure the difficulty by comparing probable errors.

It is now possible to state with some definiteness the law of difficulty for free rhythmic action. Let T be the natural period and let its probable error—that is, its difficulty—be P. It has already been observed (SCIENCE, 1896, N. S., IV., 535), that any other larger or smaller period (slower or faster beating) will be more difficult than the natural one and will have a larger probable error. Thus any interval t will have a probable error p which is greater than P, regardless of whether t is larger or smaller than T.

Three years ago (SCIENCE, as above) I promised a complete expression for this law. Continued observations during this time enable me to give an idea of its general form. The results observed can be fairly well expressed by the law

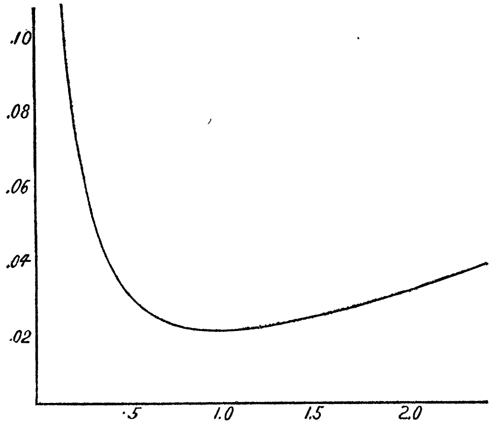
$$p = P\left(1 + c\frac{[t - T]^2}{t}\right)$$

in which T is the natural period, P the probable error for T, t any arbitrary period, p the probable error for t and c a personal constant.

This may be called the law of difficulty in free rhythmic action. A curve expressing the equation for $T = 1.0^s$, $P = 0.02^s$ and $c = 1$ is given in the figure.

It will be noticed that periods differing but little from the natural one are not much more difficult and that the difficulty increases more rapidly for smaller than for larger periods.

In plotting this curve I have assumed unity as the value for all personal constants. The personal constants will undoubtedly vary for different persons, for different occasions and for different forms



of action; an investigation is now in progress with the object of determining some of them.

In case it is desired to know what periods are of a difficulty 2, 3, ···, n times that of T, a table of values for p may be drawn up in the usual way and that value for t sought for (with interpolation) which gives for p a value 2, 3, ···, n times as great. Thus, in a table for the above example it is found that the periods 0.38^s and 2.6^s are twice as difficult.

This law can be stated in another form which is of special interest to the psychologist. To the person beating-time a period of 0 is just as far removed from his natural period as one of ∞; both are infinitely impossible. The objective scale does not express this fact; objectively a period of 0 is as different from a period of 1^s as a period of 2^s would be. Similar considerations hold good for the lesser periods; the scale by which the mind estimates periods is different from their objective scale. This difference may be expressed by asserting that the following relations exist between the two:

$$x = c\frac{(t - T)^2}{t}$$

where x is the measure on the mental scale, T the natural period, t any other period, and c a personal constant. By this formula the various periods may be laid off according to their mental differences from the natural period. Every difference from the natural period is mentally a positive matter. With the mental scale the law of difficulty becomes

$$p = P(1 + cx)$$

where p and P are the probable errors for t and T respectively, x is the measure on the mental scale and c is a personal constant. This is the equation of a straight line. The law states that the difficulty of any arbitrary period is directly proportional to its mental difference from the natural period. This is the statement which I tried to make in the note published in SCIENCE, 1896, N. S., IV., 535.

This law of difficulty as depending on the period is, of course, only one of the laws of free rhythmic action. It is quite desirable that other laws of difficulty and of frequency should be determined. For example, observations on ergograph experiments tend to show that the irregularity and the natural period both change with the weight moved; they also change with the extent of the movement.

Such a series of well established laws might be useful in regulating various activities to the best advantage. It is already recognized that it is most profitable to allow soldiers on the march to step in their natural periods; it is also known that on the contrary sudden and tense exertion is favored by changing the free rhythmic action into regulated action by marching in step and to music. More definite knowledge might perhaps be gained concerning the most profitable adjustments of the rhythm and extent of movement in bicycle-riding to the person's natural period; at present only average relations are followed in the adjustment of crank-length, gear and weight to bicycle-riders, individual and sex differences not being fully compensated. Other examples will suggest themselves.

Not only does every simple activity have its own natural rhythms; combinations of activities have rhythms that are derived from the simpler ones. In fact, it may be said that the individual, as a totality, is subjected to a series of large rhythms for his general activity (*e. g.*, yearly, monthly, weekly, daily, and so on), and also to a series of smaller rhythms for his special activities. The natural periods do not always correspond with the enforced periods. The daily rhythm is unquestionably too slow for some persons and too rapid for others; the unavoidable enforcement of the 24-hour period works a loss to all who would naturally vary from it, and diminishes the total amount of work that could be produced by them. For large numbers of brain-workers the 24-hour period is too long; for many of them the natural period is probably about 18 hours. Although about one-quarter of the day is not efficiently used, there is little relief in splitting up the day into parts, because (1) the 12-hour period would be naturally even less advantageous than the 24-hour one, and (2) the rhythm of the environment cannot be made to fit.

The progress of civilization and the changes in life are undoubtedly tending to shorten the natural period from 24 hours by encouraging a greater discharge of energy at shorter intervals. Since the 24-hour rhythm is a fixed one, there must be a constant effort at adjustment in this respect by those individuals most susceptible to the new influences. The survival of the fittest will, of course, tend to keep the natural rhythm not far from the 24-hour period.

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